



## Status and perspectives of $\sin 2\alpha$ measurements

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In the neutral  $B$  meson system, it is possible to measure the CKM angle  $\alpha$  using the decay mode  $b \rightarrow u\bar{u}d$  in the presence of pollution from gluonic  $b \rightarrow d$  penguin decays. Here the recent status of the measurements of  $CP$ -violating asymmetry parameters using time-dependent analyses in  $B^0 \rightarrow \pi^+\pi^-$  and  $B^0 \rightarrow \rho\pi$  decays and the perspectives of a  $\sin 2\alpha$  measurement are presented.

### 1 Introduction

In 1973, Kobayashi and Maskawa (KM) proposed a model where  $CP$  violation is accommodated as an irreducible complex phase in the weak-interaction quark mixing matrix [ 1]. Recent measurements of the  $CP$ -violating asymmetry parameters  $\sin 2\beta$  ( $= \sin 2\phi_1$ ) [ 2] by the Belle [ 3] and BaBar [ 4] Collaborations established  $CP$  violation in the neutral  $B$  meson system. Measurements of other  $CP$ -violating asymmetry parameters provide important tests of the KM model. Any mode with a contribution from  $b \rightarrow u\bar{u}d$  is a possible source of measurement of the Cabibbo-Kobayashi-Maskawa (CKM) angle  $\alpha$  ( $= \phi_2$ ). Here the recent status of the measurements of  $CP$ -violating asymmetry parameters using time-dependent analyses in  $B^0 \rightarrow \pi^+\pi^-$  and  $B^0 \rightarrow \rho\pi$  decays [ 5] and the perspectives of a  $\sin 2\alpha$  measurement are presented.

### 2 $B^0 \rightarrow \pi^+\pi^-$ decays

The  $B^0 \rightarrow \pi^+\pi^-$  decay is one of the important modes for the measurement of  $\sin 2\alpha$ . The KM model predicts  $CP$ -violating asymmetries in the time-dependent rates for  $B^0$  and  $\bar{B}^0$  decays to a common  $CP$  eigenstate,  $f_{CP}$ . In the decay chain  $\Upsilon(4S) \rightarrow B^0\bar{B}^0 \rightarrow f_{CP}f_{\text{tag}}$ , in which one of the  $B$  mesons decays at time  $t_{CP}$  to  $f_{CP}$  and the other decays at time  $t_{\text{tag}}$  to a final state  $f_{\text{tag}}$  that distinguishes between  $B^0$  and  $\bar{B}^0$ , the  $B^0 \rightarrow \pi^+\pi^-$  decay rate has a time-dependence given by

$$\mathcal{P}_{\pi\pi}^q(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [1 + q \cdot \{S_{\pi\pi} \sin(\Delta m_d \Delta t) - C_{\pi\pi} \cos(\Delta m_d \Delta t)\}], \quad (1)$$

where  $\tau_{B^0}$  is the  $B^0$  lifetime,  $\Delta m_d$  is the mass difference between the two  $B^0$  mass eigenstates,  $\Delta t = t_{CP} - t_{\text{tag}}$ , and the  $b$ -flavor charge  $q = +(-1)$  when the tagging  $B$  meson is a  $B^0(\bar{B}^0)$ . The  $CP$ -violating asymmetry parameters  $S_{\pi\pi}$  and  $C_{\pi\pi}$  ( $= -\mathcal{A}_{\pi\pi}$ ) [ 6] defined in Eq. (1) are expressed as

$$C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad S_{\pi\pi} = \frac{2\text{Im}\lambda_{\pi\pi}}{1 + |\lambda_{\pi\pi}|^2}, \quad (2)$$

where  $\lambda_{\pi\pi}$  is a complex parameter that depends on both  $B^0$ - $\bar{B}^0$  mixing and on the amplitudes for  $B^0$  and  $\bar{B}^0$  decay to  $\pi^+\pi^-$ . A measurement of time-dependent  $CP$ -violating asymmetries in the mode  $B^0 \rightarrow \pi^+\pi^-$  is sensitive to direct  $CP$  violation and the CKM angle  $\alpha$ . If the decay proceeded only via a  $b \rightarrow u$  tree amplitude,  $S_{\pi\pi} = \sin 2\alpha$  and  $C_{\pi\pi} = 0$ , or equivalently  $|\lambda_{\pi\pi}| = 1$ . The situation is complicated by the possibility of significant contributions from gluonic  $b \rightarrow d$  penguin amplitudes that have a different weak phase and additional strong phases. In general,  $S_{\pi\pi}$  is given by  $\sqrt{1 - C_{\pi\pi}^2} \sin 2\alpha_{\text{eff}}$ . Here  $\alpha_{\text{eff}} - \alpha$  ( $=\theta$ ) depends on the magnitudes and relative weak and strong phases of the tree and penguin amplitudes. As a result,  $S_{\pi\pi}$  may not be equal to  $\sin 2\alpha$  and direct  $CP$  violation,  $C_{\pi\pi} \neq 0$ , may occur.

Candidate  $B$  mesons are reconstructed kinematically using two variables, the energy difference  $\Delta E \equiv E_B^{\text{cms}} - E_{\text{beam}}^{\text{cms}}$  and the beam-energy constrained mass  $M_{\text{bc}} \equiv \sqrt{(E_{\text{beam}}^{\text{cms}})^2 - (p_B^{\text{cms}})^2}$  [ 7], where  $E_{\text{beam}}^{\text{cms}}$  is the cms beam energy, and  $E_B^{\text{cms}}$  and  $p_B^{\text{cms}}$  are the cms energy and momentum of the  $B$  candidate.

Charged tracks in  $B^0 \rightarrow h^+h'^-$  candidates are identified as charged pions or kaons. Here  $h$  and  $h'$  represent a  $\pi$  or  $K$ . The Belle Collaboration uses the likelihood ratio (KID) for a particle to be a  $K^\pm$  meson, which is based on the combined information from the Aerogel Cherenkov counter and CDC  $dE/dx$  measurement. Tracks are positively identified as pions with  $\text{KID} < 0.4$  for  $B^0 \rightarrow \pi^+\pi^-$  candidates. The BaBar Collaboration uses the Cherenkov angle measurement  $\theta_c$  from a detector of internally reflected Cherenkov light. The probability density function (PDF) from the difference between measured and expected values of  $\theta_c$  is used in the extended likelihood function for the fit to extract yields and  $CP$  parameters.

Background from the process  $e^+e^- \rightarrow q\bar{q}$  continuum ( $q = u, d, s, c$ ) are suppressed by their event topology. The Belle Collaboration forms signal and background likelihood functions  $\mathcal{L}_S$  and  $\mathcal{L}_{BG}$  from a Fisher discriminant determined from six modified Fox-Wolfram moments [ 8] and the cms  $B$  flight direction with respect to the beam axis. The continuum background is reduced by impos-

ing requirements on the likelihood ratio  $LR = \mathcal{L}_S / (\mathcal{L}_S + \mathcal{L}_{BG})$  for different flavor-tagging dilution factor intervals. The BaBar Collaboration uses the angle  $\theta_S$  between the sphericity axis of the  $B$  candidate and the sphericity axis of the remaining particles in the cms frame, and cut on  $|\cos \theta_S|$ . The shapes of Fisher discriminant  $\mathcal{F}$  [9] for signal and background events are included as PDFs in the maximum likelihood fit.

Leptons, kaons, and charged pions that are not associated with the reconstructed  $B$  candidate are used to identify the flavor of the accompanying  $B$  meson.

The vertex reconstruction algorithm is the same as that used for the  $\sin 2\beta$  ( $\sin 2\phi_1$ ) analysis. The time difference  $\Delta t$  is obtained from the measured distance between the  $z$  positions along the beam direction of the  $B_{\pi\pi}^0$  and  $B_{\text{tag}}^0$  decay vertices and the boost factor  $\beta\gamma$  of the  $e^+e^-$  system.

Fig. 1 and Fig. 2 show distributions of  $\Delta E$  for events enhanced in signal  $\pi^+\pi^-$  and  $K^\pm\pi^\pm$  decays from the Belle Collaboration [10] and the BaBar Collaboration [11], respectively. The Belle and BaBar Collaborations obtained the following results using an unbinned maximum likelihood fit based on  $85 \times 10^6$  and  $88 \times 10^6 B\bar{B}$  pairs, respectively,:

$$C_{\pi\pi} = -0.77 \pm 0.27 \pm 0.08, \quad S_{\pi\pi} = -1.23 \pm 0.41^{+0.08}_{-0.07} \quad (\text{Belle}),$$

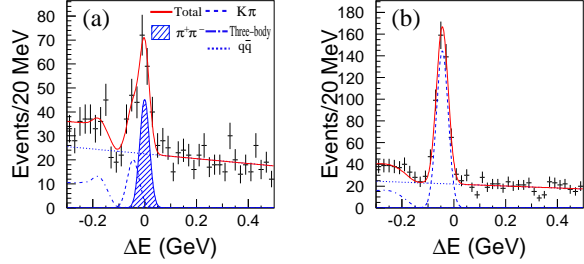
$$C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.04, \quad S_{\pi\pi} = -0.02 \pm 0.34 \pm 0.05 \quad (\text{BaBar}).$$

The first and the second errors correspond to statistical and systematic errors, respectively, unless otherwise stated. The average values of  $C_{\pi\pi}$  and  $S_{\pi\pi}$  are

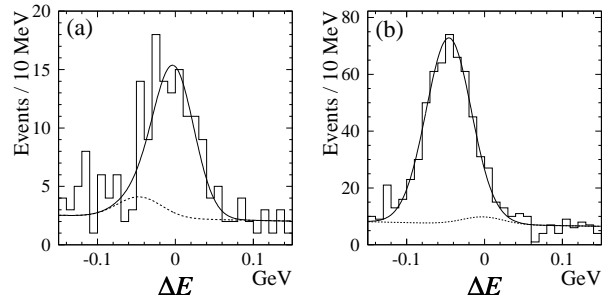
$$C_{\pi\pi} = -0.49 \pm 0.19, \quad S_{\pi\pi} = -0.47 \pm 0.26,$$

and the difference of the results between the Belle and BaBar Collaborations is  $2.2\sigma$  [12]. In Fig. 3 and Fig. 4 the  $\Delta t$  distributions for events enhanced in signal  $B^0 \rightarrow \pi^+\pi^-$  decays are shown for the Belle Collaboration [10] and the BaBar Collaborations [11], respectively.

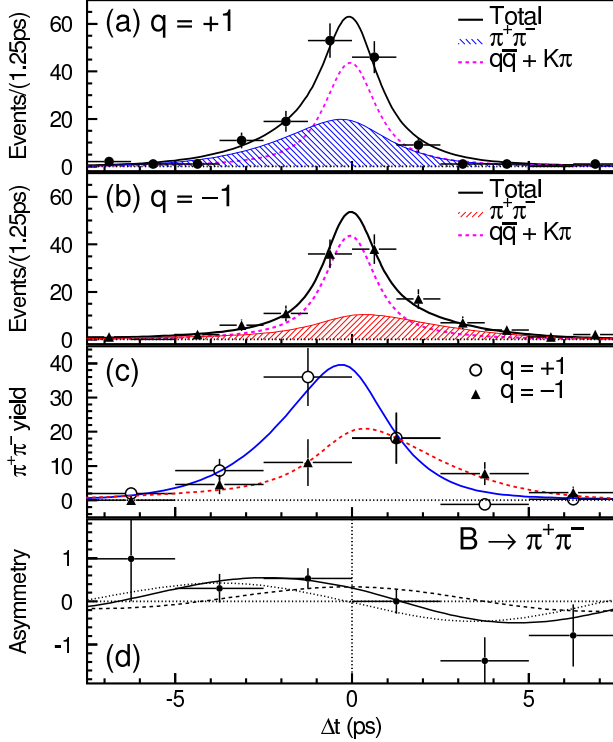
Fig. 5 shows the two-dimensional confidence regions in the  $\mathcal{A}_{\pi\pi}(= -C_{\pi\pi})$  vs.  $S_{\pi\pi}$  plane using the Feldman-Cousins frequentist approach [13]. In order to form confidence intervals, the  $\mathcal{A}_{\pi\pi}$  and  $S_{\pi\pi}$  distributions of the results of fits to MC pseudo-experiments for various input values of  $\mathcal{A}_{\pi\pi}$  and  $S_{\pi\pi}$  are used for the Belle result, and the obtained errors of  $\mathcal{A}_{\pi\pi}$  and  $S_{\pi\pi}$  are used for the BaBar result. The case that  $CP$  symmetry is conserved,  $\mathcal{A}_{\pi\pi} = S_{\pi\pi} = 0$ , is ruled out at the 99.93% confidence level (C.L.), equivalent to  $3.4\sigma$  significance for Gaussian errors from the Belle result. More data is necessary to clarify the difference between the Belle result and the BaBar result.



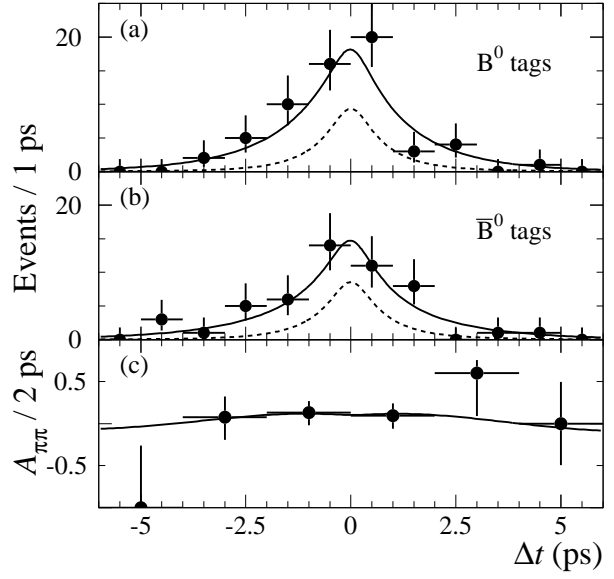
**Figure 1.**  $\Delta E$  distributions in the  $M_{bc}$  signal region for (a)  $B^0 \rightarrow \pi^+\pi^-$  candidates and (b)  $B^0 \rightarrow K^+\pi^-$  candidates with  $LR > 0.825$  from the Belle Collaboration. The sum of the signal and background functions is shown as a solid curve. The solid curve with hatched area represents the  $\pi^+\pi^-$  component, the dashed curve represents the  $K^+\pi^-$  component, the dotted curve represents the continuum background, and the dot-dashed curve represents the charmless three-body  $B$  decay background component.



**Figure 2.**  $\Delta E$  distributions for events enhanced in signal (a)  $\pi^+\pi^-$  and (b)  $K^\pm\pi^\pm$  candidates from the BaBar Collaboration. Solid curves represent projections of the maximum likelihood fit, dashed curves represent  $q\bar{q}$  and  $\pi\pi \leftrightarrow K\pi$  cross-feed background.



**Figure 3.** The raw, unweighted  $\Delta t$  distributions for  $B^0 \rightarrow \pi^+\pi^-$  candidates with  $LR > 0.825$  in the signal region from the Belle Collaboration: (a) candidates with  $q = +1$ , i.e. the tag side is identified as  $B^0$ ; (b) candidates with  $q = -1$ ; (c)  $B^0 \rightarrow \pi^+\pi^-$  yields after background subtraction; (d) the  $CP$  asymmetry for  $B^0 \rightarrow \pi^+\pi^-$  after background subtraction. In Figs. (a) through (c), the solid curves show the results of the unbinned maximum likelihood fit to the  $\Delta t$  distributions of the whole  $B^0 \rightarrow \pi^+\pi^-$  candidates. In Fig. (d), the dashed (dotted) curve is the contribution from the cosine (sine) term.



**Figure 4.** Distributions of  $\Delta t$  for events enhanced in signal  $\pi\pi$  decays from the BaBar Collaboration: the tag side is identified as (a)  $B^0$  or (b)  $\bar{B}^0$ , and (c) the asymmetry as a function of  $\Delta t$ . Solid curves represent projections of the maximum likelihood fit, dashed curves represent the sum of  $q\bar{q}$  and  $K\pi$  background events.

The decay amplitudes for  $B^0$  and  $\bar{B}^0$  to  $\pi^+\pi^-$  can be written by using the  $c$ -convention notation [ 14]:

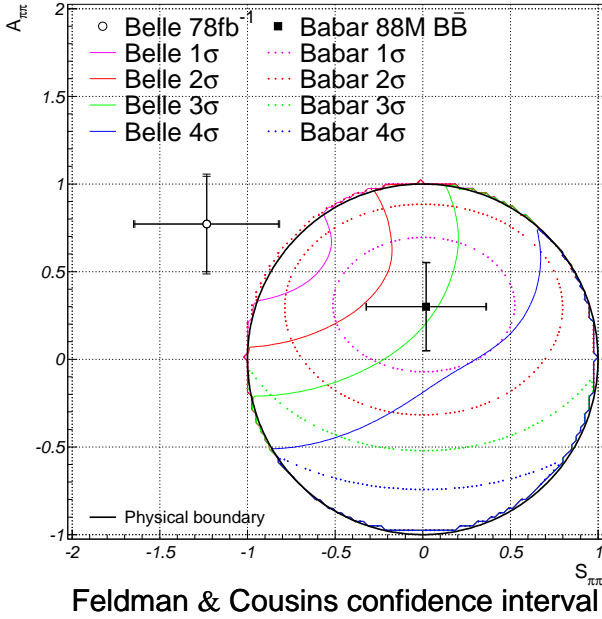
$$\begin{aligned} A(B^0 \rightarrow \pi^+\pi^-) &= -(|T|e^{i\delta_T}e^{i\phi_3} + |P|e^{i\delta_P}), \\ A(\bar{B}^0 \rightarrow \pi^+\pi^-) &= -(|T|e^{i\delta_T}e^{-i\phi_3} + |P|e^{i\delta_P}), \end{aligned} \quad (3)$$

where  $T$  and  $P$  are the amplitudes for the tree and penguin graphs and  $\delta_T$  and  $\delta_P$  are their strong phases. The expressions for  $\mathcal{S}_{\pi\pi}$  and  $\mathcal{A}_{\pi\pi}$  are

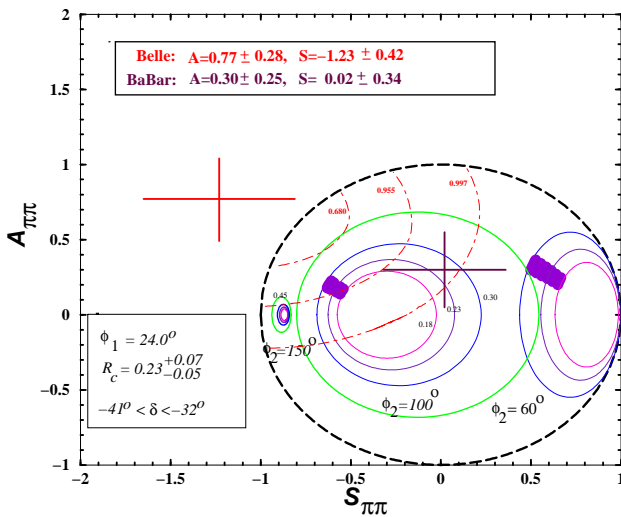
$$\begin{aligned} \mathcal{S}_{\pi\pi} &= [\sin 2\phi_2 + 2|P/T| \sin(\phi_1 - \phi_2) \cos \delta - |P/T|^2 \sin 2\phi_1] / \mathcal{R}_{\pi\pi}, \\ \mathcal{A}_{\pi\pi} &= -[2|P/T| \sin(\phi_2 + \phi_1) \sin \delta] / \mathcal{R}_{\pi\pi}, \\ \mathcal{R}_{\pi\pi} &= 1 - 2|P/T| \cos \delta \cos(\phi_2 + \phi_1) + |P/T|^2, \end{aligned} \quad (4)$$

where the strong phase difference  $\delta \equiv \delta_P - \delta_T$ . When  $\mathcal{A}_{\pi\pi}$  is positive and  $0^\circ < \phi_1 + \phi_2 < 180^\circ$ ,  $\delta$  is negative. Fig. 6 shows the two-dimensional confidence regions in the  $\mathcal{A}_{\pi\pi}$  vs.  $\mathcal{S}_{\pi\pi}$  plane together with the pQCD prediction [ 15] for various values of  $\phi_2$  ( $= \alpha$ ). Fig. 7 shows predictions for  $\mathcal{C}_{\pi\pi}$  ( $= -\mathcal{A}_{\pi\pi}$ ) and  $\mathcal{S}_{\pi\pi}$  for several analysis steps with experimental and theoretical constraints [ 16]. In Fig. 8, the interpretation of the confidence regions of  $\mathcal{A}_{\pi\pi}$  vs.  $\mathcal{S}_{\pi\pi}$  is shown in terms of  $\phi_2$  and  $\delta$  for the Belle data [ 10]. The range of  $\phi_2$  that corresponds to the 95.5% C.L. region of  $\mathcal{A}_{\pi\pi}$  and  $\mathcal{S}_{\pi\pi}$  is  $78^\circ \leq \phi_2 \leq 152^\circ$  for  $\phi_1 = 23.5^\circ$  [ 17] and  $0.15 \leq |P/T| \leq 0.45$  [ 18]. The result is in agreement with

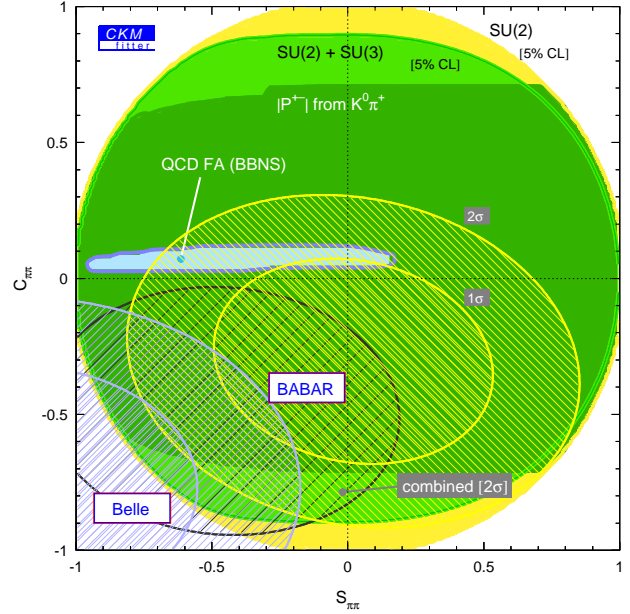
constraints on the unitarity triangle from other measurements [ 19]. Other interpretations for the current results can be found in ref. [ 16].



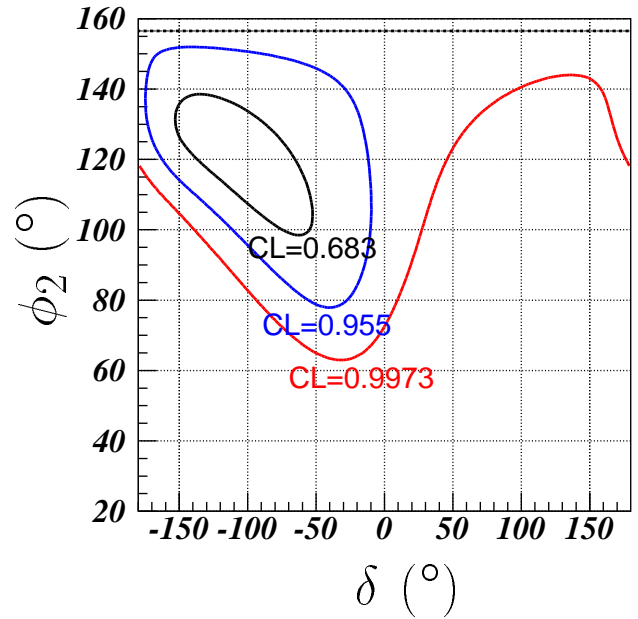
**Figure 5.** Confidence regions for  $\mathcal{A}_{\pi\pi}$  and  $S_{\pi\pi}$  from the Belle and BaBar results.



**Figure 6.** Plot of  $\mathcal{A}_{\pi\pi}$  versus  $S_{\pi\pi}$  for various values of  $\phi_2$  with  $\phi_1=24.0^\circ$ ,  $0.18 < R_c < 0.30$ , and  $-41^\circ < \delta < -32^\circ$  in the pQCD method. Here  $R_c = |P/T|$ . Dark areas are allowed regions in the pQCD method for different  $\phi_2$  values. The results of  $\mathcal{A}_{\pi\pi}$  and  $S_{\pi\pi}$  from the Belle and BaBar Collaborations and the confidence regions from the Belle Collaboration are also shown.



**Figure 7.** Predictions for  $C_{\pi\pi}$  ( $= -\mathcal{A}_{\pi\pi}$ ) and  $S_{\pi\pi}$  for several analysis steps with experimental and theoretical constraints. The Belle and BaBar results are shown.



**Figure 8.** The region for  $\phi_2$  and  $\delta$  which corresponds to the 68.3%, 95.5%, and 99.73% C.L. regions of  $\mathcal{A}_{\pi\pi}$  and  $S_{\pi\pi}$  from the Belle result in Fig. 6.  $\phi_1 = 23.5^\circ$  and  $|P/T| = 0.45$ . The horizontal dashed line corresponds to  $\phi_2 = 180^\circ - \phi_1$ .

Using isospin relations [20], we can constrain the difference,  $\theta$  between  $\alpha_{eff}$  and  $\alpha$ . From the central values of the recent world average values of the branching ratios of  $B^0 \rightarrow \pi^+\pi^-$ ,  $B^+ \rightarrow \pi^+\pi^0$  and the 90% C.L. upper limit on the  $B^0 \rightarrow \pi^0\pi^0$  branching ratio [21] together with  $C_{\pi\pi}$ , the upper limit on  $\theta$  is  $54^\circ$ .

### 3 $B^0 \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ decays

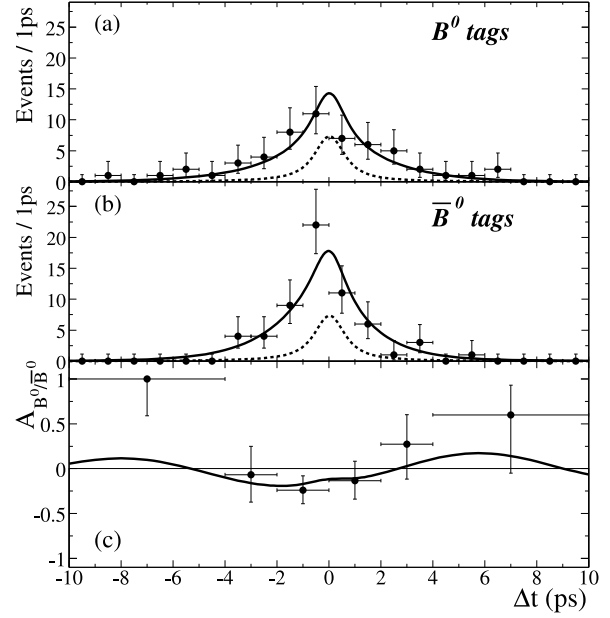
In principle, the CKM angle  $\alpha$  can be measured in the presence of penguin contributions using a full Dalitz plot analysis of the final state. However, there are difficulties of combinatorics and lower efficiency in three-body topology with  $\pi^0$  and large backgrounds from misreconstructed signal events and other decays. In order to extract  $\alpha$  cleanly, data with large statistics are required.

Unlike  $B^0 \rightarrow \pi^+\pi^-$  decay,  $B^0 \rightarrow \rho^+\pi^-$  decay is not a  $CP$  eigenstate, and four flavor-charge configurations ( $B^0(\bar{B}^0) \rightarrow \rho^\pm\pi^\mp$ ) must be considered. Following a quasi-two-body approach [22], the analysis is restricted to the two regions of the  $\pi^\pm\pi^0h^\pm$  Dalitz plot ( $h = \pi$  or  $K$ ) that are dominated by  $\rho^\pm h^\mp$ . The decay rate is given by

$$f_q^{\rho^\pm h^\mp}(\Delta t) = (1 \pm A_{CP}^{\rho h}) \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \times [1 + q \cdot ((S_{\rho h} \pm \Delta S_{\rho h}) \sin(\Delta m_d \Delta t) - q \cdot (C_{\rho h} \pm \Delta C_{\rho h}) \cos(\Delta m_d \Delta t))], \quad (5)$$

where  $\Delta t = t_{\rho h} - t_{tag}$  as the time interval between the decay of  $B_{\rho h}^0$  and that of the other  $B^0$  meson in the event,  $B_{tag}^0$ .

The time- and flavor-integrated charge asymmetries  $A_{CP}^{\rho\pi}$  and  $A_{CP}^{\rho K}$  measure direct  $CP$  violation. For the  $\rho\pi$  mode, the quantities  $S_{\rho\pi}$  and  $C_{\rho\pi}$  parameterize mixing-induced  $CP$  violation related to the CKM angle  $\alpha$ , and flavor-dependent direct  $CP$  violation, respectively.  $\Delta C_{\rho\pi}$  describes the asymmetry between the rates  $\Gamma(B^0 \rightarrow \rho^+\pi^-) + \Gamma(\bar{B}^0 \rightarrow \rho^-\pi^+)$  and  $\Gamma(B^0 \rightarrow \rho^-\pi^+) + \Gamma(\bar{B}^0 \rightarrow \rho^+\pi^-)$ .  $\Delta S_{\rho\pi}$  is related to the strong phase difference between the amplitudes contributing to  $B^0 \rightarrow \rho\pi$  decays. One finds the relations  $S_{\rho\pi} \pm \Delta S_{\rho\pi} = \sqrt{1 - (C_{\rho\pi} \pm \Delta C_{\rho\pi})^2} \sin(2\alpha_{eff}^\pm \pm \delta)$ , where  $2\alpha_{eff}^\pm = \arg[(q/p)(\bar{A}_{\rho\pi}^\pm/A_{\rho\pi}^\mp)]$ ,  $\delta = \arg[A_{\rho\pi}^-/A_{\rho\pi}^+]$ ,  $\arg[q/p]$  is the  $B^0$ - $\bar{B}^0$  mixing phase, and  $A_{\rho\pi}^+$  ( $\bar{A}_{\rho\pi}^+$ ) and  $A_{\rho\pi}^-$  ( $\bar{A}_{\rho\pi}^-$ ) are the transition amplitudes of the processes  $B^0(\bar{B}^0) \rightarrow \rho^+\pi^-$  and  $B^0(\bar{B}^0) \rightarrow \rho^-\pi^+$ , respectively. The angles  $\alpha_{eff}^\pm$  are equal to  $\alpha$  if contributions from penguin amplitudes are absent. For the self-tagging  $\rho K$  mode, the values of the four time-dependent parameters are  $C_{\rho K} = 0$ ,  $\Delta C_{\rho K} = -1$ ,  $S_{\rho K} = 0$ , and  $\Delta S_{\rho K} = 0$ . The results on direct  $CP$  violation can be



**Figure 9.** Time distributions for events selected to enhance the  $\rho\pi$  signal tagged as (a)  $B^0$ -tag and (b)  $\bar{B}^0$ -tag, and (c) time-dependent asymmetry between  $B^0$ -tag and  $\bar{B}^0$ -tag from the BaBar Collaboration [23]. The solid curve is a likelihood projection of the fit result. The dashed line is the sum of  $B$ - and continuum-background contributions.

expressed using the asymmetries

$$A_{+-} = \frac{N(\bar{B}_{\rho\pi}^0 \rightarrow \rho^+\pi^-) - N(B_{\rho\pi}^0 \rightarrow \rho^-\pi^+)}{N(\bar{B}_{\rho\pi}^0 \rightarrow \rho^+\pi^-) + N(B_{\rho\pi}^0 \rightarrow \rho^-\pi^+)} = \frac{A_{CP}^{\rho\pi} - C_{\rho\pi} - A_{CP}^{\rho\pi} \cdot \Delta C_{\rho\pi}}{1 - \Delta C_{\rho\pi} - A_{CP}^{\rho\pi} \cdot C_{\rho\pi}} \quad (6)$$

$$A_{-+} = \frac{N(\bar{B}_{\rho\pi}^0 \rightarrow \rho^-\pi^+) - N(B_{\rho\pi}^0 \rightarrow \rho^+\pi^-)}{N(\bar{B}_{\rho\pi}^0 \rightarrow \rho^-\pi^+) + N(B_{\rho\pi}^0 \rightarrow \rho^+\pi^-)} = \frac{A_{CP}^{\rho\pi} + C_{\rho\pi} + A_{CP}^{\rho\pi} \cdot \Delta C_{\rho\pi}}{1 + \Delta C_{\rho\pi} + A_{CP}^{\rho\pi} \cdot C_{\rho\pi}} \quad (7)$$

With a data sample of 89 million  $B\bar{B}$  pairs [23], the BaBar Collaboration found  $428_{-33}^{+34}(\text{stat})$   $\rho\pi$  and  $120_{-20}^{+21}(\text{stat})$   $\rho K$  events and the following measurements of the  $CP$  violation parameters are obtained:

$$\begin{aligned} A_{CP}^{\rho\pi} &= -0.18 \pm 0.08 \pm 0.03, \\ C_{\rho\pi} &= +0.36 \pm 0.18 \pm 0.04, \\ S_{\rho\pi} &= +0.19 \pm 0.24 \pm 0.03. \end{aligned}$$

For the other parameters in the description of the  $B^0(\bar{B}^0) \rightarrow \rho\pi$  decay-time dependence,

$$\begin{aligned} \Delta C_{\rho\pi} &= +0.28 \pm 0.19 \pm 0.04, \\ \Delta S_{\rho\pi} &= +0.15 \pm 0.25 \pm 0.03, \end{aligned}$$

are found. For the asymmetries  $A_{+-}$  and  $A_{-+}$ , which probe direct  $CP$  violation,

$$\begin{aligned} A_{+-} &= -0.62^{+0.24}_{-0.28} \pm 0.06, \\ A_{-+} &= -0.11^{+0.16}_{-0.17} \pm 0.04, \end{aligned}$$

are measured. The raw time-dependent asymmetry in the tagging categories dominated by kaons and leptons is shown in Fig. 9.

## 4 Prospects

Table 1 shows the expected error on  $CP$ -violating parameters in  $B^0 \rightarrow \pi^+\pi^-$  and  $B^0 \rightarrow \rho^+\pi^-$  decays with accumulated luminosities of  $140 \text{ fb}^{-1}$ ,  $400 \text{ fb}^{-1}$ ,  $3000 \text{ fb}^{-1}$  ( $3 \text{ ab}^{-1}$ ), and  $30000 \text{ fb}^{-1}$  ( $30 \text{ ab}^{-1}$ ) in the future.

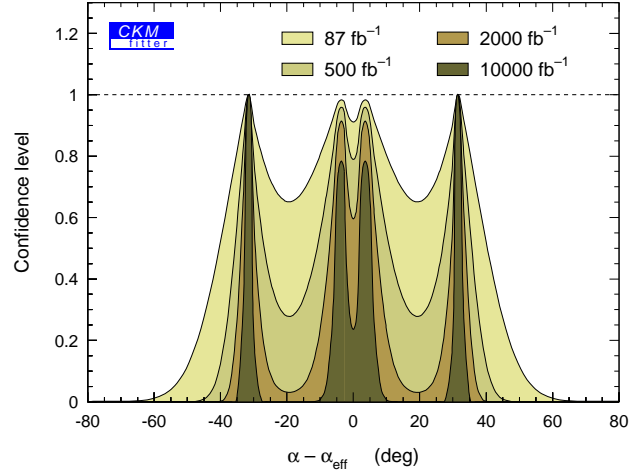
Fig. 10 shows the prospects of  $\alpha_{eff} - \alpha$  for the ICHEP'02 central values of the  $B \rightarrow \pi\pi$  branching ratios and the central values of  $S_{\pi\pi}$  and  $C_{\pi\pi}$  from the BaBar measurement at luminosities of the current and future B-factories ( $87 \text{ fb}^{-1}$ ,  $500 \text{ fb}^{-1}$ ,  $2 \text{ ab}^{-1}$ ,  $10 \text{ ab}^{-1}$ ). The inner and outer borders can be obtained from the isospin analysis when  $B^0 \rightarrow \pi^0\pi^0$  flavors are tagged and not tagged, respectively. Only a luminosity of around  $10 \text{ ab}^{-1}$  allows to separate the solutions.

For  $B \rightarrow \rho\pi$  decays, the isospin analysis is not feasible yet with the present statistics of the  $B$  factories. In [24], the projections into the future full  $SU(2)$  analysis was demonstrated. If the branching fraction of  $B^0 \rightarrow \rho^0\pi^0$  is below the experimental sensitivity, a strong constraint on  $\alpha$  is expected above luminosity of around  $2 \text{ ab}^{-1}$ . In this workshop, theoretical problems such as form factors and  $\sigma$  meson [25], and experimental problems for several sources of backgrounds were pointed out.

Detailed discussions can be found in [27] for  $B^0 \rightarrow \pi^+\pi^-$  and in [24] for  $B^0 \rightarrow \rho\pi$  at this workshop.

parameters	$140 \text{ fb}^{-1}$	$400 \text{ fb}^{-1}$	$3 \text{ ab}^{-1}$	$30 \text{ ab}^{-1}$
$\mathcal{A}_{\pi\pi}$	0.21	0.13	0.05	0.02
$S_{\pi\pi}$	0.31	0.19	0.07	0.03
$A_{CP}^{\rho\pi}$	0.07	0.04	0.02	0.008
$C_{\rho\pi}$	0.14	0.09	0.03	0.013
$S_{\rho\pi}$	0.18	0.11	0.04	0.014

**Table 1.** The errors of  $CP$ -violating parameters in  $B^0 \rightarrow \pi^+\pi^-$  and  $B^0 \rightarrow \rho^+\pi^-$  decays at  $140 \text{ fb}^{-1}$ ,  $400 \text{ fb}^{-1}$ ,  $3 \text{ ab}^{-1}$ , and  $30 \text{ ab}^{-1}$ , assuming that statistical and systematic errors are proportional to  $1/\sqrt{\mathcal{L}}$  and  $1/\sqrt[4]{\mathcal{L}}$ , respectively. Here  $\mathcal{L}$  is accumulated luminosity.



**Figure 10.**  $\alpha_{eff} - \alpha$  for the ICHEP'02 central values of the  $B \rightarrow \pi\pi$  branching fractions and the central values of  $S_{\pi\pi}$  and  $C_{\pi\pi}$  from the BaBar measurement at luminosities of the current and future B-factories ( $87 \text{ fb}^{-1}$ ,  $500 \text{ fb}^{-1}$ ,  $2 \text{ ab}^{-1}$ , and  $10 \text{ ab}^{-1}$ ).

## 5 Summary

In summary, the Belle and BaBar Collaborations obtain the following measurements of the  $CP$ -violating asymmetry parameters in  $B^0 \rightarrow \pi^+\pi^-$  decays:

$$\begin{aligned} \mathcal{A}_{\pi\pi} &= +0.77 \pm 0.27 \pm 0.08, & S_{\pi\pi} &= -1.23 \pm 0.41^{+0.08}_{-0.07} \\ & & & \text{(Belle),} \\ \mathcal{A}_{\pi\pi} &= +0.30 \pm 0.25 \pm 0.04, & S_{\pi\pi} &= -0.02 \pm 0.34 \pm 0.05 \\ & & & \text{(BaBar),} \end{aligned}$$

The following measurements of the  $CP$ -violating asymmetry parameters in  $B^0 \rightarrow \rho\pi$  decays using a quasi two-body analysis are obtained by the BaBar Collaboration:

$$\begin{aligned} A_{CP}^{\rho\pi} &= -0.18 \pm 0.08 \pm 0.03, \\ C_{\rho\pi} &= +0.36 \pm 0.18 \pm 0.04, & \Delta C_{\rho\pi} &= +0.28 \pm 0.19 \pm 0.04, \\ S_{\rho\pi} &= +0.19 \pm 0.24 \pm 0.03, & \Delta S_{\rho\pi} &= +0.15 \pm 0.25 \pm 0.03. \end{aligned}$$

For the asymmetries  $A_{+-}$  and  $A_{-+}$ , which probe direct  $CP$  violation,

$$A_{+-} = -0.62^{+0.24}_{-0.28} \pm 0.06, \quad A_{-+} = -0.11^{+0.16}_{-0.17} \pm 0.04,$$

are obtained.

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2.  $\phi_1 (= \beta) \equiv \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$ ,  $\phi_2 (= \alpha) \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$ , and  $\phi_3 (= \gamma) \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$ .
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6.  $C_{\pi\pi} = -\mathcal{A}_{\pi\pi}$ . The BaBar Collaboration uses  $C_{\pi\pi}$  and the Belle Collaboration uses  $\mathcal{A}_{\pi\pi}$ . Usually the partial-rate asymmetry or direct  $CP$  violation parameter is defined by  $[N(\bar{B} \rightarrow \bar{f}) - N(B \rightarrow f)]/[N(\bar{B} \rightarrow \bar{f}) + N(B \rightarrow f)]$ . Its sign is consistent with the sign of  $\mathcal{A}_{\pi\pi}$ . Here  $B$  represents either a  $B^0$  or  $B^+$  meson,  $f$  represents a flavor-specific final state, and  $\bar{B}^0$  and  $\bar{f}$  are their conjugates.
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